Absorbed Dose in Radioactive Media

Chapter 5

F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

Outline

- General dose calculation considerations, absorbed fraction
- Radioactive disintegration processes and associated dose deposition
  - Alpha disintegration
  - Beta disintegration
  - Electron-capture transitions
  - Internal conversion
- Summary

Introduction

- We are interested in calculating the absorbed dose in radioactive media, applicable to cases of
  - Dose within a radioactive organ
  - Dose in one organ due to radioactive source in another organ
- If conditions of CPE or RE are satisfied, dose calculation is straightforward
- Intermediate situation is more difficult but can be handled at least in approximations

Radiation equilibrium

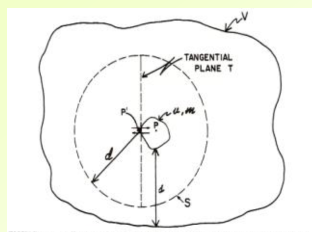


FIGURE 4.1. Radiation equilibrium. Extended volume V contains a homogeneous medium and a homogeneous isotropic source distribution. Radiation equilibrium will exist in the smaller internal volume v if the maximum distance of penetration (d) of primary rays plus their secondaries is less than the minimum separation (s) of v from the boundary of V. Neutrinos are ignored. (See text.)

a. The atomic composition of the medium is homogeneous
b. The density of the medium is homogeneous
c. The radioactive source is uniformly distributed
d. No external electric or magnetic fields are present

Charged-particle equilibrium

- Each charged particle of a given type and energy leaving the volume is replaced by an identical particle of the same energy entering the volume
- Existence of RE is sufficient condition for CPE
- Even if RE does not exist CPE may still exist (for a very large or a very small volume)

Limiting cases

- Emitted radiation typically includes both photons (longer range) and charged particles (shorter range)
- Assume the conditions for RE are satisfied

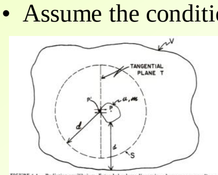


FIGURE 4.1. Radiation equilibrium. Extended volume V contains a homogeneous medium and a homogeneous isotropic source distribution. Radiation equilibrium will exist in the smaller internal volume v if the maximum distance of penetration (d) of primary rays plus their secondaries is less than the minimum separation (s) of v from the boundary of V. Neutrinos are ignored. (See text.)

- Consider two limited cases based on the size of the radioactive object

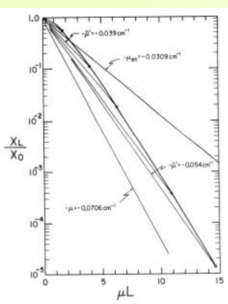
Limiting cases: small object

- A radioactive object V having a mean radius not much greater than the maximum charged-particle range d
- CPE is well approximated at any internal point *P* that is at least a distance *d* from the boundary of *V*
- If *d*« l/μ for the γ-rays, the absorbed dose *D* at P approximately equals to the energy per unit mass of medium that is given to the charged particles in radioactive decay (less their radiative losses)
- The photons escape from the object and are assumed not to be scattered back by its surroundings

Limiting cases: large object

- A radioactive object with mean radius » l/μ for the most penetrating γ-rays
- RE is well approximated at any internal point P that is far enough from the boundary of V so γ-ray penetration through that distance is negligible
- The dose at P will then equal the sum of the energy per unit mass of medium that is given to charged particles plus γ-rays in radioactive decay

Limiting cases: large object



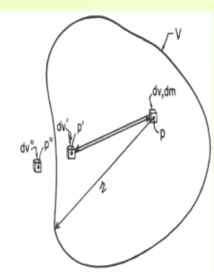
- Deciding upon a maximum γ-ray “range” for this case requires quantitative criterion
  - For primary beam (μ) only less than 0.1% through 7 mean free paths
  - Taking into account scatter, (broad beam geometry, $\overline{\mu}'$) will increase the required object size to satisfy the attenuation objectives (e.g. 16 mean free paths to achieve 0.1% penetration

Absorbed fraction

- An *intermediate*-size radioactive object V
- Dose at P will then equal the sum of the energy per unit mass of medium that is given to charged particles plus dose from γ-rays
- To estimate the dose from γ-rays define absorbed fraction:

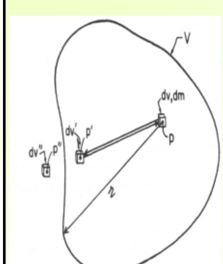
$$AF = \frac{\gamma\text{ - ray radiant energy absorbed in target volume}}{\gamma\text{ - ray radiant energy emitted by source}}$$

Intermediate-size radioactive object



- Consider volume V filled by a homogeneous medium and a uniformly distributed γ-source
- The volume may be surrounded by
  - Case 1: infinite homogeneous medium identical to V, but non-radioactive (an organ in the body)
  - Case 2: Infinite vacuum (object in air)

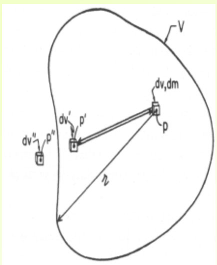
Case 1



- Reciprocity theorem: energy spent in *dv* due to the source in *dv'*:

  $\varepsilon_{dv',dv} = \varepsilon_{dv,dv'}$ and $\varepsilon_{dv,V} = \varepsilon_{V,dv}$
- No source in *dv''*
- Define $\mathcal{R}_{dv}$ as the expectation value of the γ-ray radiant energy emitted by the source in *dv*, and $\overline{\varepsilon}_{dv,V}$ the part of that energy that is spent in *V*, then the absorbed fraction with respect to source *dv* and target *V* is

Case 1 continued



- The absorbed fraction with respect to source *dv* and target *V* is :

$$AF_{dv,V} = \frac{\bar{\varepsilon}_{dv,V}}{\bar{R}_{dv}} = \frac{\bar{\varepsilon}_{V,dv}}{\bar{R}_{dv}}$$

source, target

- Estimates reduction in absorbed dose relative to RE condition
- For very small radioactive objects ($V \rightarrow dv$) this absorbed fraction approaches zero; for an infinite radioactive medium it equals unity

Case 1 continued

- Reduction in the absorbed dose due to γ-rays energy escaping from V
- Can estimate AF using mean effective attenuation coefficient $\bar{\mu}'$ for γ-rays energy fluence through a distance *r* in the medium

$$AF_{dv,V} = \frac{1}{4\pi} \int_{\theta=0}^{\pi} \int_{\beta=0}^{2\pi} \left(1 - e^{-\bar{\mu}' r}\right) \sin\theta d\theta d\beta$$

- For poly-energetic sources have to find an average value of the absorbed fraction

Case 1 example

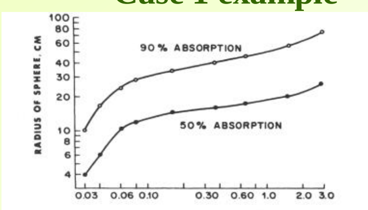
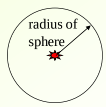


FIGURE 5.2. Radius of unit-density tissue sphere needed to absorb 50% and 90% of the emitted photon energy from a central point source in an infinite homogeneous medium. (After Brownell et al., 1968.) Reproduced with permission of the authors and The Society of Nuclear Medicine.



- Dose calculations published in MIRD reports
- The larger the radius, the lower the energy – the closer to RE condition (*AF*=1)

Case 2

Radioactive object V surrounded by vacuum

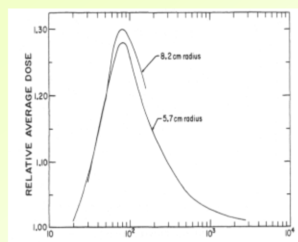


FIGURE 5.3. Ratio of average absorbed doses in uniformly radioactive tissue spheres with/without surrounding non-radioactive tissue medium. Lower curve: 780-g (5.7-cm-radius) sphere; upper curve: 2.3-kg (8.2-cm-radius) sphere. (Data from Ellett, 1968.)

- More difficult to calculate
- The reciprocity theorem is only approximate due to the lack of backscattering
- Dose is lower than in Case 1

Case 2

- To obtain a crude estimate of the dose at some point *P* within a uniformly γ-active homogeneous object, it may suffice to obtain the average distance $\bar{r}$ from the point to the surface of the object by

$$\bar{r} = \frac{1}{4\pi} \int_{\theta=0}^{\pi} \int_{\beta=0}^{2\pi} r \sin\theta \, d\theta \, d\beta$$

- Then one may employ $\mu_{en} = \bar{\mu}'$ in the straight-ahead approximation to obtain

$$AF_{dv,V} \cong 1 - e^{-\mu_{en}\bar{r}}$$

Radioactive disintegration processes

- Radioactive nuclei tend to undergo transformations to a more stable state through expulsion of energetic particles

	ΔZ	ΔA
α-particle	−2	−4
β^--particle†	+1	0
β^+-particle†	−1	0
γ-ray	0	0

- The total mass, energy, momentum, and electric charge are conserved
- Energy equivalent of the rest mass:
  - 1 amu=1/12 of the mass of $^{12}_{6}$C nucleus=931.50 MeV
  - 1 electron mass=0.51100 MeV

Alpha disintegration

- Occurs mainly in heavy nuclei
- Example: decay of radium to radon, presented by the mass-energy balance equation

$$\underset{\text{parent}}{{}^{226}_{88}\text{Ra}} \xrightarrow[\tau_{1/2}=1602\,\text{y}]{} \underset{\text{daughter}}{{}^{222}_{86}\text{Rn}} + {}^{4}_{2}\text{He} + 4.78\text{ MeV}$$

- Each of the elemental terms represents the rest mass of a neutral atom of that element

Alpha disintegration

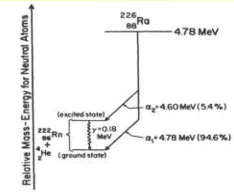


FIGURE 5.4. Atomic energy level diagram for ${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn}$, showing branching in the modes of disintegration. Note that the rest-mass energy of the ${}^{222}_{86}\text{Rn}$ atom has been augmented by that of the ${}^{4}_{2}\text{He}$ atom so that the vertical scale need not be expanded to include the loss of energy equivalent to the α-particle mass (≅4 × 938 MeV).

- Two branches are available for disintegration
- Kinetic + quantum energy released is 4.78MeV
- After the α-particle slows down it captures two electrons from its surroundings, becoming a neutral He atom

Absorbed dose from α- disintegration

- Take into account both branches through the average branching ratios
- For radium average kinetic energy given to charged particles per disintegration

$$E_\alpha = 0.946 \times 4.78\,\text{MeV} + 0.054 \times 4.6\,\text{MeV} = 4.77\,\text{MeV}$$

- For CPE condition in a small (1 cm) radium-activated object: D=4.77 x n MeV/g, where n – mumber of disintegration per gram of the matter
- For RE condition (large object): D=4.78 x n MeV/g, includes γ-ray energy

Beta disintegration

- Nuclei having excess of neutrons typically emit an electron, β^- particle; atomic number Z is increased by 1
- Nuclei having excess of protons typically emit positron, β^+ particle; atomic number Z is decreased by 1
- Nucleus is left in an exited state, emitting one or more γ-rays
- Example:

$${}^{32}_{15}\text{P} \xrightarrow[\tau_{1/2}=14.3\,\text{d}]{} {}^{32}_{16}\text{S} + \beta^- + {}^{0}_{0}\nu + 1.71\,\text{MeV}$$

Beta disintegration

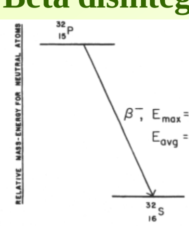


FIGURE 5.6. Atomic energy-level diagram for ${}^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S}$ β^--disintegration.

- Kinetic energy 1.71 MeV is shared between β^- and neutrino
- Charge balance is realized through initially ion of ${}^{32}_{16}\text{S}^+$ capturing an electron

Beta disintegration

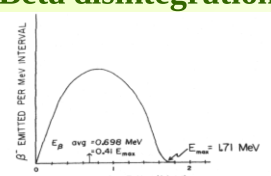


FIGURE 5.5. β^--ray spectrum emitted from ${}^{32}_{15}\text{P}$. The average energy of the particles is 0.694 MeV. The abscissa may be alternatively labeled "1.71 − neutrino energy (MeV)" to make the curve indicate the neutrino spectrum.

- Average kinetic energy of the β^- or β^+ particle is typically 0.3-0.4 E_{max}; 1/3 E_{max} is often used for purposes of roughly estimating absorbed dose
- Neutrino is not included in dose estimates due to almost zero rest mass energy and no charge

Beta disintegration

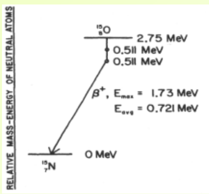


FIGURE 5.7. Atomic energy-level diagram for β^+-emission by $^{15}_{8}$O. The vertical line segments below the $^{15}_{8}$O energy level indicate the loss of the rest-mass energy of the β^+ and the corresponding valence electron released by the atom during disintegration.

- In β^+ disintegration valence electron is emitted
- Example: $^{15}{}_{8}$O -> $^{15}{}_{7}$N

$$^{15}_{8}\text{O} \underset{\tau_{1/2}=122\,\text{s}}{\rightarrow} {}^{15}_{7}\text{N} + \underbrace{\text{e}^- + \beta^+}_{} + {}^{0}_{0}\nu + 1.73\,\text{MeV}$$

positron annihilation yields $2m_0c^2$=1.022 MeV

Absorbed dose from β- disintegration

- Under CPE condition D=nE_{avg} MeV/g for n disintegration per gram of medium
- Any additional contributions to energy deposition due to γ-rays must be included for RE condition
- Radiative losses by β-rays, such as bremsstrahlung and in-flight annihilation, are ignored

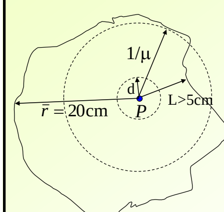
Example 5.1

- Uniformly distributed β- and γ-ray source
- The rest-mass loss is spent
  - half in 1 -MeV γ -ray production and
  - half in β^- -decay, for which E_{max}=5 MeV and E_{avg}=2 MeV
- The point of interest P is located >5 cm inside the boundary of the object, at an average distance $\bar{r}$ = 20 cm from the boundary.
- μ_{en}=0.0306 cm^{-1} and μ = 0.0699 cm $^{-1}$ for the γ -rays
- A total energy of 10^{-2} J converted from rest mass in each kg of the object
- Estimate the absorbed dose at P

Example 5.1

- For β-ray E_{max}=5 MeV corresponds to maximum particle range (Appendix E) d ~2.6 cm << 1/μ=14.3 cm
- CPE exists at P, therefore dose due β-rays:

$$D_\beta = E_{avg} \times \frac{1}{2} \times 10^{-2}\,\text{J/kg} = 10^{-2}\,\text{J/kg}$$



- For γ-ray 20cm is not >> 1/μ=14.3 cm
- RE does not exists at P, therefore have to use absorbed fraction:

$$AF \cong 1 - e^{-\mu_{en}\bar{r}} = 0.46$$

$$D_\gamma = 0.46 \times (1/2) \times 10^{-2}\,\text{J/kg} = 2.3 \times 10^{-3}\,\text{J/kg}$$

$$\text{Dtot} = (2.3 + 10) \times 10^{-3}\,\text{J/kg} = 1.23 \times 10^{-2}\,\text{J/kg}$$

Example 5.2

- What is the absorbed dose rate (Gy/h) at the center of a sphere of water 1 cm in radius, homogeneously radioactivated by $^{32}{}_{15}$P, with 6 X 10^5 disintegrations per second occurring per gram of water? (Assume time constancy.)

Example 5.2

- E_{max}=1.71 MeV corresponds to maximum particle range of ~0.8 cm < 1 cm
- CPE condition
- Absorbed dose rate: $\dot{D} = \dot{N} \times E_{avg}$

$$\dot{D} = 6 \times 10^5 \frac{\text{dis}}{g\,\text{sec}} \times 0.694 \frac{\text{MeV}}{\text{dis}}$$

$$= 4.164 \times 10^5 \frac{\text{MeV}}{g\,\text{sec}} \times 3600 \frac{\text{s}}{\text{hr}} \times 1.602 \times 10^{-10} \frac{\text{Gy}}{\text{MeV/g}}$$

$$= 2.4\,\text{Gy/h}$$

Electron-capture transitions

- Parent nucleus captures its own atomic electron from K-shell (~90% probability) or L-shell (~10% probability) and emits monoenergetic neutrino
- Resulting shell vacancy is filled with electron from a higher orbit, leading to emission of a fluorescence x-ray
- Process competing with β^+ disintegrations

Electron-capture transitions

- Example: ${}^{22}_{11}Na$ -> ${}^{22}_{10}Ne$ with half-life for both branches of 2.60 years
  - β^+ branch

$$ {}^{22}_{11}\mathrm{Na} \underset{\tau_{1/2}=2.60\mathrm{y}}{\rightarrow} {}^{22}_{10}\mathrm{Ne} + \underbrace{\mathrm{e}^- + \beta^+}_{2\mathrm{m}_0=1.022\ \mathrm{MeV}} + {}^{0}_{0}\nu + 0.546\,\mathrm{MeV\,(k.e.)} $$

  - EC branch

$$ {}^{22}_{11}\mathrm{Na} \underset{\tau_{1/2}=2.60\mathrm{y}}{\rightarrow} {}^{22}_{10}\mathrm{Ne} + \underbrace{{}^{0}_{0}\nu + \mathrm{E_b}}_{1.568\ \mathrm{MeV}} + 1.275\,\mathrm{MeV\,(E\gamma)} $$

Electron-capture transitions

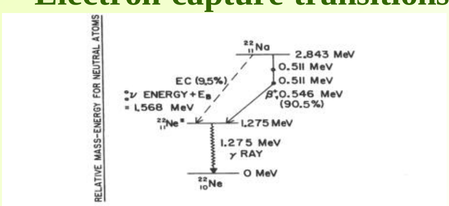


FIGURE 5.8. Atomic energy-level diagram for β^+ and EC disintegration of ${}^{22}_{11}Na \xrightarrow[\tau_{1/2}=2.60\,y]{} {}^{22}_{10}Ne$. $(E_\beta)_{max} = 0.546$ MeV; $\bar{E}_\beta = 0.216$ MeV.

- Binding energy for K-shell E_b~ 1 keV
- For electron capture to occur the minimum atomic mass decrease of $2m_0$ between the parent and daughter nuclei is required to supply β^+ with kinetic energy

Absorbed dose for EC process

- Most of the energy is carried away by neutrino
- The only available energy for dose deposition comes from electron binding term E_b, which is very small compare to that of neutrino

Internal conversion

- An excited nucleus can impart its energy directly to its own atomic electron, which then escapes with the net kinetic energy of $h\nu$-E_b ($h\nu$ is the excitation energy)
- No photon is emitted in this case
- Process competing with γ-ray emission
- Internal conversion coefficient is the ratio of N_e/N_γ

Internal conversion

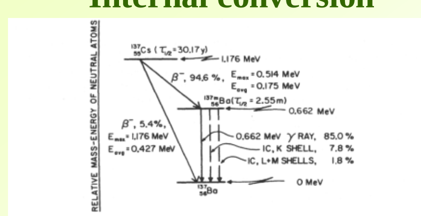


FIGURE 5.9. Atomic energy-level diagram for ${}^{137}_{55}Cs \xrightarrow[\tau_{1/2}=30.17y]{} {}^{137}_{56}Ba$, illustrating competition between γ-ray emission and internal conversion. Percentages all refer to disintegrations of parent atoms of ${}^{137}_{55}Cs$. $(e_K/\gamma) = 0.0916$, $K/(L + M + \ldots) = 4.41$.

- Example: ${}^{137}_{55}Cs$ -> ${}^{137}_{56}Ba$

Absorbed dose for internal conversion

- If IC occurs in competition with γ-ray emission, it results in increase in absorbed dose in small objects (CPE condition) due to release of electron locally depositing the energy

$$E_{IC} = h\nu - E_b$$

- In addition electron binding energy is contributed to the dose unless it escapes as a fluorescence x-ray

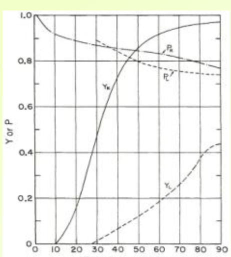
Absorbed dose for internal conversion

- If the fraction $p = 1 - AF$ of these fluorescence x-rays escape, then the energy contributed to dose per IC event under CPE condition

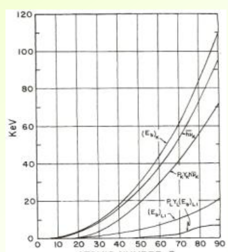
$$f_{IC} = h\nu - p_K Y_K h\bar{\nu}_K - p_L Y_L h\bar{\nu}_L$$

- Using straight-ahead approximation $p \cong e^{-\mu_{en}\bar{r}}$
- Values of fluorescence yield $Y_{K,L}$ and the mean emitted x-ray energies $h\nu_{K,L}$ are tabulated

Fluorescence data



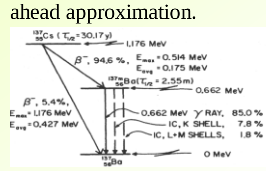
Fluorescence yield for K and L shells



Electron binding energies and mean fluorescence x-ray energies, K and L shells

Example 5.5

- A sphere of water 10 cm in diameter contains a uniform source of ^{137}Cs undergoing 10^3 disintegration per g s. What is the absorbed dose at the center, in grays, for a 10-day period, due only to the decay of $^{137m}_{56}Ba$? Use the mean-radius straight-ahead approximation.



For γ-ray of 0.662 MeV in water μ_{en}=0.0327 cm^{-1}

Example 5.5

- First check RE condition: $1/\mu$=30.6 cm >> r=5
- Find absorbed fraction $AF \cong 1 - e^{-0.0327\times 5} = 0.151$
- Dose in 10 days=8.64×10^5 s is

$$D_\gamma = 10^3 \frac{dis}{g\ s} \times 0.85 \frac{\gamma - rays}{dis} \times 0.662 \frac{MeV}{\gamma - ray}$$
$$\times 1.602\times10^{-10} \frac{Gy}{MeV/g} \times 8.64\times10^5 s \times AF$$
$$= 1.17\times10^{-2} Gy$$

Example 5.5

- For the K-shell conversion process we need Y_K=0.90, $h\bar{\nu}_K$ = 0.032 MeV, and μ_{en}=0.13 cm^{-1} for 0.032 MeV. Then $p_K \cong e^{-\mu_{en}\bar{r}} = e^{-0.13\times5} = 0.52$ and the dose contribution is

$$D_{IC}^K = 10^3 \frac{dis}{g\ s} \times 0.078 \frac{IC(K)}{dis} \times (h\nu - p_K Y_K h\bar{\nu}_K) \frac{MeV}{IC(K)}$$
$$\times 1.602\times10^{-10} \frac{Gy}{MeV/g} \times 8.64\times10^5 s$$
$$= 1.080(0.662 - 0.52\times0.90\times0.032)\times10^{-2} Gy$$
$$= 6.99\times10^{-3} Gy$$

Example 5.5

- Similarly, for the L+M+…-shell conversion process we need Y_K=0.90, $h\bar{\nu}_L = E_b^L$= 6 keV, and μ_{en}=24 cm^{-1} for 6 keV. Then $p_L \cong e^{-24\times 5} = 0$ and the corresponding dose contribution

$$D_{IC}^{L} = 10^3 \frac{dis}{g\ s} \times 0.078 \frac{IC(L)}{dis} \times (h\nu - p_L Y_L h\bar{\nu}_L) \frac{MeV}{IC(L)}$$
$$\times 1.602\times 10^{-10} \frac{Gy}{MeV/g} \times 8.64\times 10^{5}\, s$$
$$= 1.65\times 10^{-3}\, Gy$$

- The total absorbed dose is $D_{tot} = D_\gamma + D_{IC}^K + D_{IC}^L = 2.03\times 10^{-2}\, Gy$

Summary

- General approach to dose calculation within and outside of distributed radioactive source
- Radioactive disintegration processes and calculation of absorbed dose
  - Alpha disintegration
  - Beta disintegration
  - Electron-capture transitions
  - Internal conversion